Karmaveer Bhaurao Patil University, Satara

Syllabus for

B. Sc. II Mathematics

Under

Faculty of Science and Technology

(As per NEP 2020)

With effect from Academic Year 2025-2026

1. STRUCTURE OF COURSE:

As per NEP-2020 (2.0)

Level	Sem.	Courses					OE	VSC/ SEC	AEC/ VEC/ IKS	OJT/ FP/ CEP/ CC/ RP	Total
		Course I		Course II		Course III					
		Major		Minor							
		T	P	T	P						
5.0	III	Major V (2) Major VI (2)	Major P III (2)	Minor V (2) Minor VI (2)	Minor P III (2)	--	OE III (2)	VSC I (P) (2) SEC I (P) (2)	AEC 1 (2) IKS 2 (2)	--	22
5.0	IV	Major VII (2) Major VIII (2)	Major P IV (2)	Minor VII (2) Minor VIII (2)	Minor P IV (2)	--	OE IV (2)	VSC II (P) (2) SEC II (P) (2)	AEC 2 (2) VEC 2 (2)	--	22

2. COURSE TITLE:

1) Third Semester

Sr. No.	Subject	Theory				Practical		
		Course No. & Course Code	Title of Course	No. of lectures per week	Credits	Course No. & Course Code	No. of Practical Per week	Credits
1.	Major	BMT 231	Introduction to Real Analysis	2	2	BMP 233: Practical Based on BMT 231 & BMT 232	1	2
		BMT 232	Basics of Algebra	2	2			
2.	Minor	BMT 234	Vector Calculus	2	2	BMP 236: Practical Based on BMT 234 & BMT 235	1	2
		BMT 235	Introduction to Linear Algebra	2	2			
3.	OE	--	--	--	--	BMPOE 3	1	2
4.	VSC	--	--	--	--	BMPVSC 1	1	2
5.	SEC	--	--	--	--	BMPSEC 1	1	2
6.	AEC	BMTAEC 1	English	2	2	--	--	--
7.	IKS	BMTIKS 2	Vedic Mathematics	2	2	--	--	--

2) Fourth Semester

Sr. No.	Subject	Theory				Practical		
		Course Code	Title of Course	No. of lectures per week	Credits	Course Code & Title	No. of Practical Per week	Credits
1.	Major	BMT 241	Real Analysis	2	2	BMP 243: Practical Based on BMT 241 & BMT 242	1	2
		BMT 242	Abstract Algebra	2	2			
2.	Minor	BMT 244	Numerical Analysis	2	2	BMP 246: Practical Based on BMT 244 & BMT 245	1	2
		BMT 245	Discrete Mathematics	2	2			
3.	OE	--	--	--	--	BMPOE 4	1	2
4.	VSC	--	--	--	--	BMPVSC 2	1	2
5.	SEC	--	--	--	--	BMPSEC 2	1	2
6.	AEC	BMTAEC 2	English	2	2	--	--	--
7.	VEC	BMTVEC 2	Environmental Awareness with Mathematics	2	2	--	--	--

3. EVALUATION STRUCTURE:

Semester III (5.0)

Course	Course Category	Course Code	Internal Evaluation			ESE	Total Marks	Credits
			CCE - I	CCE - II	CCE - III			
Major	T	BMT 231	10	10	10	30	50	02
	T	BMT 232	10	10	10	30	50	02
	P	BMP 233	20			30	50	02
Minor	T	BMT 234	10	10	10	30	50	02
	T	BMT 235	10	10	10	30	50	02
	P	BMP 236	20			30	50	02
OE	P	BMPOE 3 (IDS)	20			30	50	02
OE	T	BMTOE 3 (For Humanities)	10	10	10	30	50	02
VSC	P	BMPVSC 1	20			30	50	02
SEC	P	BMPSEC 1 Practical	20			30	50	02
AEC I	T	BMTAEC 1	10	10	10	30	50	02
IKS II	T	BMTIKS 2	10	10	10	30	50	02
						Total	**550**	**22**

Semester IV (5.0)

Course	Course Category	Course Code	Internal Evaluation			ESE	Total Marks	Credits
			CCE - I	CCE - II	CCE - III			
Major	T	BMT 241	10	10	10	30	50	02
	T	BMT 242	10	10	10	30	50	02
	P	BMP 243	20			30	50	02
Minor	T	BMT 244	10	10	10	30	50	02
	T	BMT 245	10	10	10	30	50	02
	P	BMP 246	20			30	50	02
OE	P	BMPOE 4 (IDS)	20			30	50	02
OE	T	BMTOE 4 (For Humanities)	10	10	10	30	50	02
VSC	P	BMPVSC 2	20			30	50	02
SEC	P	BMPSEC 2 Practical	20			30	50	02
AEC II	T	BMTAEC 2	10	10	10	30	50	02
VEC II	T	BMTVEC 2	10	10	10	30	50	02
						Total	**550**	**22**

VSC: Vocational Skill Course; SEC: Skill Enhancement Course; AEC: Ability Enhancement Course; OE: Open Elective; VEC: Value Education Course; IKS: Indian Knowledge System T: Theory; P: Practical

CONTINOUS AND COMPHERENSIVE EVALUATION (CCE) STRUCTURE:

- Internal evaluation of theory papers shall carry a weightage of 40%.
- For the internal evaluation of theory papers, departments have the flexibility to adopt a variety of assessment methods, including class tests, assignments, book review, tutorials, seminars, case studies, fieldwork, project work, quizzes, and similar tools for CCE I and CCE II.
- CCE III shall consist of multiple-choice questions.
- Each CCE will carry 10 marks for 2-credit theory papers and 20 marks for 4-credit theory
- papers, and the best two scores will be considered for final mark entry.
- Internal evaluation of practical papers shall carry a weightage of 50%.
- For practical examinations, internal evaluation will be based on student performance, attendance, journal records, and viva voce.

4. OTHER FEATURES:

A) LIBRARY:

Reference books, Textbooks, journals, and Periodicals are available in Institute and Departmental Library. (Separate reference lists are attached along with the respective course syllabus)

B) EQUIPMENT:

a) Computer, LCD projector, visualizer, smart board

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: Major	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 231 Introduction to Real Analysis

Course Objectives: By the end of the course, students should be able to……

1. define and explore set operations, relations, and the properties of functions (including injective, surjective, bijective, and inverse mappings).
2. distinguish the concepts of finite, infinite, countable, and uncountable sets, and prove the uncountability of real numbers.
3. establish the Completeness Property of the real number system, focusing on bounds, the Archimedean property, and the density of irrationals.
4. introduce the formal theory of sequences and subsequences of real numbers, including the rigorous definition of limits and convergence.

Course Outcomes: By the end of the course, students will be able to……

1. apply the properties of sets and functions to prove relationships between subsets, direct/inverse images, and functional compositions
2. analyze the cardinality of mathematical sets to classify them as countably infinite or uncountable using established theorems.
3. evaluate subsets of real numbers to determine their supremum and infimum by utilizing the Completeness and Archimedean properties.
4. determine the limits and convergence of real number sequences using formal mathematical definitions and bounding theorems.

Module	Title and Contents	Hrs
Module - 1:	**Sets and Functions** 1.1 Sets 1.1.1 Operations on sets, Cartesian product of sets, Relation 1.2 Functions 1.2.1 Definitions: Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function 1.2.2 Theorem: If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$ then $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$. 1.2.3 Theorem: If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$ then $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$ 1.2.4 Theorem: If $f: A \to B$ and $X \subseteq A, Y \subseteq A$ then $f(X \cup Y) = f(X) \cup f(Y)$ 1.2.5 Theorem: If $f: A \to B$ and $X \subseteq A, Y \subseteq A$ then $f(X \cap Y) \subseteq f(X) \cap f(Y)$ 1.2.6 Definitions: Injective, Surjective and Bijective function (1-1 correspondence), Inverse function. 1.2.7 Theorem: Composition of two bijective functions is a bijective function.	7

Module - 2:	**Countable Sets** 2.1 Countable set: 2.1.1 Definitions: Finite sets, Infinite sets, Countable Sets, Uncountable Sets. 2.1.2 Examples of Countable sets: Set of Natural numbers, Set of Integers, Cartesian product of Countable sets 2.1.3 Theorem: Countable union of countable set is countable. 2.1.4 Theorem: Set of Rational numbers is countable. 2.1.5 Theorem: Any subset of countable set is countable. 2.1.6 Theorem: The closed interval $[0,1]$ is uncountable. 2.1.7 Theorem: The set of all real numbers is uncountable. 2.1.8 Theorem: Every infinite set has a countably infinite subset. 2.1.9 Examples	8
Module - 3:	**Completeness Property of $\mathbb{R}$** 3.1 Definitions: Lower bound, Upper bound of a subset of $\mathbb{R}$, Bounded set, Supremum $(l.u.b)$, Infimum $(g.l.b)$. 3.2 Least Upper Bound Axiom [Completeness Property of $\mathbb{R}$]: 3.3 Theorem (Archimedean Property): If $x \in \mathbb{R}$ then there exists $n_x \in \mathbb{N}$ such that $x \leq n_x$. 3.3.1 Corollary: If $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ then $\inf S = 0$ 3.3.2 Corollary: If $t > 0$ then there exist $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$ 3.3.3 Corollary: If $y > 0$ then there exist $n_y \in \mathbb{N}$ such that $n_y - 1 < y < n_y$ 3.4 Theorem: There exists a positive real number x such that $x^2 = 2$ 3.4.1 Corollary: If x and y are real numbers with $x < y$ then there exist an irrational number z such that $x < z < y$ 3.5 Intervals 3.5.1 Characterization Theorem: If S is a subset of $\mathbb{R}$ that contains at least two points and has the property if $x, y \in S$ and $x < y$ then the closed interval $[x, y] \subseteq S$ then S is an interval.	7
Module - 4:	**Sequence of Real Numbers** 4.1 Sequence and Subsequence: 4.4.1 Definition of Sequence and Subsequence 4.4.2 Examples 4.2 Limit of a Sequence 4.2.1 Definition 4.2.2 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} S_n = L$ then $L \geq 0$. 4.2.3 Examples 4.3 Convergent Sequence	8

Reference Books:

1. D. Somasundaram and B Choudhary, First Course in Mathematical Analysis, 8th Reprint, Narosa Publishing house New, Delhi, 2013

2. P. K. Jain and S. K. Kaushik, An Introduction to Real Analysis, 1st Edition, S. Chand & Company Ltd. New Delhi, 2000
3. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley India Pvt. Ltd, 2016
4. Shanti Narayan and M. D. Raisinghania, Elements of Real Analysis, 15th Edition, S. Chand & Company Ltd. New Delhi, 2014

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: Major	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 232 Basics of Algebra

Course Objectives: By the end of the course, students should be able to……

1. explain the fundamental principles of integer divisibility, including the Euclidean Algorithm, greatest common divisors, and the Unique Factorization Theorem
2. illustrate the concept of mathematical relations, their pictorial and matrix representations, and the structural properties of equivalence and partial order relations.
3. demonstrate the algebraic properties of congruence relations and their application in solving linear congruences via the Chinese Remainder Theorem and Fermat's theorem.
4. examine the foundational algebraic structures of semigroups, monoids, and groups, focusing on group properties, element orders, and permutations.

Course Outcomes: By the end of the course, students will be able to……

1. apply divisibility rules, the Euclidean algorithm, and the fundamental theorem of arithmetic to compute greatest common divisors and solve integer-based problems.
2. analyze sets and mappings to classify relations (reflexive, symmetric, transitive), compute compositions, and construct equivalence partitions
3. solve complex modular arithmetic problems and linear congruences by executing the Chinese Remainder Theorem and Fermat's theorem.
4. evaluate algebraic systems to determine if they form groups or sub-structures, and compute the order of elements and cyclic permutations within those groups.

Module	Title and Contents	Hrs
Module - 1:	**Divisibility in Integers** 1.1 Definition: Divisibility in integers 1.2 The Well Ordering Principle (Statement Only) 1.3 Properties of Divisibility 1.3.1 Definition of divisor and Multiple 1.3.2 Theorem: Let a, b, c, d be integers. Then (i) If $a \mid b$ then $a \mid bx$ (ii) If $a \mid b$ and $a \mid c$ then $a \mid bx + cy$; $\forall\, x, y \in I$ (iii) If $a \mid b$ and $b \mid c$ then $a \mid c$ (iv) If m $\neq$ 0 is in $\mathbb{Z}$ and $a \mid b \Rightarrow am \mid bm$ (v) If $a \mid b$ and $c \mid d$ then $ac \mid bd$ (vi) If $ab \mid bc$ then $a \mid c;\ (b \neq 0)$ 1.4 Theorem: Division Algorithm (Without Proof) 1.5 Greatest common divisor and least common multiple 1.5.1 Definitions: Greatest common divisor and least common multiple	7

	1.5.2 Theorem: Let a and b be two integers at least one of them not zero. Then there exists a unique greatest common divisor d of a and b. Moreover, d can be written as $d = am + bn$ for integers m and n. 1.5.3 The Euclidean Algorithm and Examples. 1.5.4 Definition: Relatively Prime 1.5.5 Euclid's lemma: For a prime number p, if $p \mid ab$ then either $p \mid a$ or $p \mid b$. 1.6 Unique Factorization Theorem or Fundamentals Theorem of Arithmetic.	
Module - 2:	**Relation** 2.1 Relation 2.1.1 Definitions: Cartesian Product, Relation, Binary Relation, Inverse Relation. 2.1.2 Examples on 2.1.1. 2.2 Pictorial Representation of Relation 2.2.1 Co-ordinate Diagram 2.2.2 Arrow Diagram 2.2.3 Matrix Representation 2.2.4 Directed Graph 2.2.5 Examples on 2.2.1 to 2.2.4 2.3 Compositions of Relations 2.3.1 Definition: Composition of Relations 2.3.2 Theorem: Let A, B, C and D be sets. Let R: A⟶ B, S: B⟶ C, T: C⟶ D be Relation then $R \circ (S \circ T) = (R \circ S) \circ T$. 2.4 Types of Relations 2.4.1 Definitions: Reflexive, Symmetric, Antisymmetric and transitive. 2.4.2 Examples on 2.4.1 2.4.3 Theorem: Let R be relation on set A. Then R^{∞} is the smallest transitive relation on A that contain R. 2.5 Equivalence relation and partitions 2.5.1 Theorem: Let R be an equivalence relation on set A. Then quotient set A/R forms a partition of A. 2.5.2 Theorem: Let $\{A_i\}, i \in I$ be partition of a set A. Then there exists an equivalence relation R on the set A such that quotient set A / R is the given partition $\{A_i\}, i \in I$ on A. 2.6 Partial order relation 2.6.1 Definition: Partial order relation. 2.6.2 Examples on 2.6.1	8
Module - 3:	**Congruence Relations** 3.1 Definition and examples 3.1.1 Theorem: Let $n > 1$ be a fixed positive integer and a be any integer then $a \equiv a \pmod{n}$. 3.1.2 Theorem: Let $n > 1$ be a fixed positive integer and a, b be any integers and if $a \equiv b \pmod{n}$ then $b \equiv a \pmod{n}$.	7

	3.1.3 Theorem: Let n >1 be a fixed positive integer and a, b, c be any integers and if a $\equiv$ b (mod n) and b $\equiv$ c (mod n) then a $\equiv$ c (mod n). 3.1.4 Theorem: Let n >1 be a fixed positive integer and a, b, c, d be any integers and if a $\equiv$ b(modn) and c $\equiv$ d(modn) then i) a + c $\equiv$ b + d (modn) and ii) ac $\equiv$ bd (modn) 3.1.5 Theorem: Let n >1 be a fixed positive integer and a, b be any integers and if a $\equiv$ b (mod n) then i) a + c $\equiv$ b + c (modn) and ii) ac $\equiv$ bc (modn). 3.1.6 Theorem: Let n >1 be a fixed positive integer and a, b be any integers and if a $\equiv$ b (mod n) then $a^k \equiv b^k$ (mod n), for any positive integer k. 3.1.7 Theorem: If $ca \equiv cb(mod\ n)$ then $a \equiv b\left(mod\frac{n}{d}\right)$; where $d = \gcd(a, b)$. 3.2 Divisibility test 3.3 Linear congruence and examples 3.4 Chinese Remainder Theorem and examples 3.5 Fermat's theorem and examples	
Module - 4:	**Groups** 4.1 Binary operation on a set 4.1.1 Definition: Binary operation on a set with illustration 4.2 Semigroup 4.2.1 Definition: Semigroup with illustration 4.3 Monoid 4.3.1 Definition: Monoid with illustration 4.4 Group 4.4.1 Definition: Group, Abelian Group, Finite Group, Infinite Group, Order of a group 4.4.2 Examples on 4.4.1 4.5 Properties of Groups 4.5.1 Theorem: If $<G,*>$ is a group, then a) Identity element in G is unique b) Every a in G has unique inverse in G. c) For every a in G, $(a^{-1})^{-1} = a$ d) For all $a, b \in G$, $(a * b)^{-1} = b^{-1} * a^{-1}$ 4.5.2 Theorem: If a, b, c are elements in a group G, then (i) a $*$ b = a $*$ c implies b = c (Left Cancellation Law) (ii) b $*$ a = c $*$ a implies b = c (Right Cancellation Law) 4.5.3 Theorem: If G is a group and $a, b \in G$, then the equations a $*$ x = b and y $*$ a = b have unique solutions $x = a^{-1} * b$ and $y = b^{-1} * a$ respectively. 4.5.4 Definition: Order of element with illustration, Properties (Without Proof) 4.6 Permutation 4.6.1 Definition with Illustration 4.6.2 Cyclic Permutation 4.6.3 Transposition, Disjoint Permutations, Even and Odd Permutations	8

Reference Books:

1. David. M. Burton, Elementary Number Theory, 7th Edition, McGraw Hill Education, 2017
2. Alberts, B., Bray, D., Hopkin, K., Johnson, A.D., Johnson, A., Lewis, J., Raff, M., Roberts, K., Walter, P. "Essential Cell Biology". New York: Garland Science, 2009
3. Seymour Hipschutz, Marehipson, Schaum's Outline, Discrete Mathematics, 3rd Edition, Tata MaGraw-Hill Publishing Company Ltd., New Delhi, July 2017 (Unit 3)
4. Shantinarayan, A Text Book of Matrices, 11th Edition, S. Chand Co., Pvt. Ltd. Raminagar, New Delhi, 2005
5. V. K. Khanna and S. K. Bhambri, A course in abstract Algebra, 5th Edition, Vikas Publishing House Private Limited, New Delhi, 2016 (Unit 4)

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: Major Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMP 233 Practical Based on BMT 231 & BMT 232

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. classify various mathematical relations (equivalence, partial order) and apply concepts of function types and compositions.
2. analyze the fundamental properties of real numbers, including set cardinality, completeness, and sequence convergence.
3. solve computational problems using divisibility properties, the Euclidean algorithm, and modular arithmetic theorems.
4. introduce basic algebraic structures, teaching students to *identify* groups and *determine* the order of mathematical elements.

Course Outcomes: By the end of the course, students will be able to……

1. illustrate and classify complex mathematical relations and compute the compositions and properties of various functions.
2. analyze the topological and algebraic properties of real numbers by determining set countability and evaluating the limits of convergent sequences.
3. apply number theory algorithms (such as the Euclidean Algorithm, Chinese Remainder Theorem, and Fermat's theorem) to solve integer and divisibility problems.
4. determine the foundational structure of algebraic groups and compute the order of their constituent elements.

Module	Title and Contents	Hrs.
Section I	1. Relation on sets and its type.	**60**
	2. Functions and its types.	
	3. Composition of functions.	
	4. Countable and uncountable sets.	
	5. Completeness of property of $\mathbb{R}$.	
	6. Limit of sequence and its examples.	
	7. Convergent sequences.	
	8. Properties of divisibility.	
	9. Euclidean Algorithm.	
	10. Pictorial Representation of Relation.	
	11. Equivalence relation.	
	12. Partial order relation.	
	13. Chinese Remainder Theorem.	
	14. Fermat's theorem.	

	15. Group & Order of an element.	

Reference Books:

1. D. Somasundaram and B Choudhary, First Course in Mathematical Analysis, 8th Reprint, Narosa Publishing house New, Delhi, 2013
2. David. M. Burton, Elementary Number Theory, 7th Edition, McGraw Hill Education, 2017
3. P. K. Jain and S. K. Kaushik, An Introduction to Real Analysis, 1st Edition, S. Chand & Company Ltd. New Delhi, 2000
4. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley India Pvt. Ltd, 2016
5. Seymour Hipschutz, Marehipson, Schaum's Outline, Discrete Mathematics, 3rd Edition, Tata MaGraw-Hill Publishing Company Ltd., New Delhi, July 2017 (Unit 3)
6. Shanti Narayan and M. D. Raisinghania, Elements of Real Analysis, 15th Edition, S. Chand & Company Ltd. New Delhi, 2014
7. Shantinarayan, A Text Book of Matrices, 11th Edition, S. Chand Co., Pvt. Ltd. Raminagar, New Delhi, 2005
8. V. K. Khanna and S. K. Bhambri, A course in abstract Algebra, 5th Edition, Vikas Publishing House Private Limited, New Delhi, 2016 (Unit 4)

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: Minor	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 234 Vector Calculus

Course Objectives: By the end of the course, students should be able to……

1. introduce vector-valued functions, parametric space curves, and their fundamental differential and integral calculus.
2. demonstrate the computation of gradients and directional derivatives and their geometric applications to tangent planes and normal lines.
3. explain the physical interpretations, mathematical properties, and operations of vector differential fields (divergence and curl).
4. equip students with the techniques of surface integration and fundamental vector integral theorems (Green's, Stokes', and Gauss Divergence).

Course Outcomes: By the end of the course, students will be able to……

1. apply differentiation and integration techniques to solve problems involving vector-valued functions and parametric curves in space.
2. compute directional derivatives and gradient vectors to construct tangent planes and normal lines for surfaces in space.
3. analyze the physical behavior and properties of vector fields by evaluating their divergence and curl.
4. evaluate line, surface, and flux integrals by selecting and applying Green's, Stokes', and Gauss Divergence theorems.

Module	Title and Contents	Hrs
Module - 1:	**Introduction to Vectors and Vector Functions** 1.1 Revision: Vector presentation and notation, Vector operation, Dot product and cross product 1.2 Vector function: Definition, Properties of vector functions 1.3 Parametric equations and curves in space 1.4 Differentiation and Integration of Vector functions	8
Module - 2:	**Gradient and Directional Derivatives** 2.1 Definition, Properties of gradients 2.2 Directional derivative and gradient vector 2.3 Application of gradients in finding tangent planes and normal lines	7
Module - 3:	**Divergence and Curl** 3.1 Divergence: Definition, Interpretation of divergence 3.2 Properties of divergence 3.3 Curl: Definition, Interpretation of curl 3.4 Properties of curl	7
Module - 4:	**Vector Integral Calculus** 4.1 Surface integral: Definition, Application of Surface integral	8

	4.2 Green's Theorem (Statement only) and its application 4.3 Stokes' Theorem (Statement only) and its application 4.4 Gauss divergence Theorem (Statement only) and its application	

Reference Books:

1. A. R. Vasishtha, Vector Calculus, Krishna Prakashan Media, 2020.
2. J. E. Marsden and A. Tromba, Vector Calculus, 6th edition, W. H. Freeman and Co. Ltd., 2011.
3. M. L. Khanna, Vector Calculus, Jaiprakash Nath and Co. Meerut, 1997.
4. M. R. Spiegel, S. Lipschutz and D. Spellman, Vector Analysis, 2nd edition, McGraw Hill Education, 2017
5. R. Gupta, Vector Calculus, Laxmi Publications, 2016.
6. S. J. Colley, Vector Calculus, 4th edition, Pearson, 2011.
7. S. Narayan and P. K. Mittal, A Textbook of Vector Calculus, 4th edition, S. Chand, 1987.

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: Minor	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 235 Introduction to Linear Algebra

Course Objectives: By the end of the course, students should be able to……

1. introduce the properties of special matrices (Hermitian/Skew-Hermitian) and the use of elementary transformations to determine matrix rank.
2. provide systematic computational methods (Gauss elimination and Gauss-Jordan) for analyzing and solving systems of linear equations.
3. apply the foundational concepts of eigenvalues and eigenvectors, alongside the practical applications of the Cayley-Hamilton Theorem.
4. explain the algebraic theory of real quadratic forms and the methodology for reducing them to their canonical forms.

Course Outcomes: By the end of the course, students will be able to……

1. apply elementary row and column transformations to reduce matrices to echelon forms and determine their rank.
2. evaluate the consistency of homogeneous and non-homogeneous linear systems and solve them utilizing Gauss elimination and Gauss-Jordan methods.
3. compute characteristic polynomials, eigenvalues, and eigenvectors, and utilize the Cayley-Hamilton theorem for advanced matrix operations.
4. analyze real quadratic forms by mathematically reducing them to canonical form to identify their signature and index.

Module	Title and Contents	Hrs
Module - 1:	**Matrices and Rank of Matrix** 1.1 Revision 1.2 Hermitian and Skew Hermitian matrices, Properties of Hermitian and Skew Hermitian matrices 1.3 Sub-matrices of Matrix, Minors of Matrix, Rank of matrix (Definition) 1.4 Echelon form of a matrix and Reduced row echelon form 1.5 Elementary transformation of a matrices, Elementary matrices, Invariance of rank under elementary transformation	7
Module - 2:	**System of Linear Homogeneous and Non-Homogeneous Equations** 2.1 Condition for consistency 2.2 Nature of the general solution 2.3 Gauss Elimination and Gauss Jordan method (using row echelon form and reduced row echelon form) 2.4 Examples	8
Module - 3:	**Eigenvalues and Eigenvectors of Matrix** 3.1 Characteristic polynomial of matrix, Characteristic equation of matrix	7

	3.2 Eigenvalue or characteristic root of matrix 3.3 Eigenvector or characteristic vector of matrix 3.4 Cayley-Hamilton Theorem (Statement only) and its application	
Module - 4:	**Quadratic Forms** 4.1 Quadratic forms 4.2 Reduction of real quadratic form 4.3 Canonical or Normal form of a real quadratic form 4.4 Signature and Index of a real quadratic form	8

Reference Books:

1. A. R. Vasishtha and A. K. Vasishtha, Matrices, Krishna Prakashan, 2014.
2. D. C. Lay, S. R. Lay and J. J. McDonald, Linear Algebra and Its Applications, 6th edition, Pearson, 2021.
3. G. Strang, Introduction to Linear Algebra, 5th edition, Wellesley-Cambridge Press, 2016.
4. K. Hoffman and R. Kunze, Linear Algebra, Pearson, 2018.
5. S. H. Friedberg, A. J. Insel and L. E. Spence, Linear Algebra, 4th edition, Pearson Education India, 2015.
6. S. Kumaresan, Linear Algebra, PHI Learning Pvt. Ltd., 2000.
7. S. Lipschutz and M. Lipson, Linear Algebra, 3rd edition, McGraw Hill Education, 2017.
8. S. Narayan and P. K. Mittal, A Textbook of Matrices, S Chand and Company, 2010.

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: Minor Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMP 236 Practical Based on BMT 234 & BMT 235

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. introduce differential operators (gradient, divergence, curl) and their applications in finding directional derivatives and tangent planes.
2. demonstrate the physical and mathematical applications of fundamental integral theorems, including Green's, Stokes', and the Divergence theorems.
3. learn the properties of special matrices, matrix rank determination, and techniques for solving systems of linear equations.
4. explore eigenvalues, eigenvectors, the Cayley-Hamilton theorem for matrix inversion, and the classification of real quadratic forms.

Course Outcomes: By the end of the course, students will be able to……

1. compute gradients, directional derivatives, divergence, and curl to analyze the geometric and physical properties of vector fields.
2. evaluate complex line, surface, and volume integrals by applying Green's, Stokes', and the Divergence theorems.
3. solve systems of linear homogeneous and non-homogeneous equations by utilizing matrix rank and structural properties.
4. analyze linear transformations by calculating eigenvalues/eigenvectors, applying the Cayley-Hamilton theorem, and reducing real quadratic forms to their canonical state.

Module	Title and Contents	Hrs.
Section I	1. Vector function.	**60**
	2. Gradients and directional derivative.	
	3. Examples on finding tangent planes and normal lines.	
	4. Properties of divergence and curl.	
	5. Application of Green's Theorem.	
	6. Application of Stokes' Theorem.	
	7. Application of Divergence Theorem.	
	8. Hermitian and Skew Hermitian matrices.	
	9. Rank of matrix.	
	10. Solution of system of m linear homogeneous equations in n unknowns.	
	11. Solution of system of m linear non-homogeneous equations in n unknowns.	
	12. Eigenvalues and Eigenvectors of matrix.	
	13. Inverse of matrix using Cayley-Hamilton Theorem.	
	14. Canonical form of real quadratic form.	

	15. Signature and Index of a real quadratic form.	

Reference Books:

1. A. R. Vasishtha, Vector Calculus, Krishna Prakashan Media, 2020.
2. A. R. Vasishtha and A. K. Vasishtha, Matrices, Krishna Prakashan, 2014.
3. D. C. Lay, S. R. Lay and J. J. McDonald, Linear Algebra and Its Applications, 6th edition, Pearson, 2021.
4. G. Strang, Introduction to Linear Algebra, 5th edition, Wellesley-Cambridge Press, 2016.
5. J. E. Marsden and A. Tromba, Vector Calculus, 6th edition, W. H. Freeman and Co. Ltd., 2011.
6. K. Hoffman and R. Kunze, Linear Algebra, Pearson, 2018.
7. M. L. Khanna, Vector Calculus, Jaiprakash Nath and Co. Meerut, 1997.
8. M. R. Spiegel, S. Lipschutz and D. Spellman, Vector Analysis, 2nd edition, McGraw Hill Education, 2017.
9. R. Gupta, Vector Calculus, Laxmi Publications, 2016.
10. S. H. Friedberg, A. J. Insel and L. E. Spence, Linear Algebra, 4th edition, Pearson Education India, 2015.

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: VSC Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMPVSC 1 Data Analysis using MATLAB

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. introduce the MATLAB environment, basic commands, and mathematical operations involving vectors, matrices, and arrays.
2. perform data handling techniques, including managing various data types, organizing tabular data, and preprocessing raw datasets.
3. demonstrate foundational data analysis methods in MATLAB, such as data smoothing, computing linear correlations, and polynomial curve fitting.
4. build algorithmic problem-solving skills using control structures (loops, decision branching), user-defined functions, and code troubleshooting techniques.

Course Outcomes: By the end of the course, students will be able to……

1. apply fundamental MATLAB commands and mathematical operators to create, manipulate, and extract data from vectors, matrices, and arrays.
2. process and organize diverse datasets (including tables) by utilizing conditional data selection and preprocessing techniques.
3. analyze experimental or real-world datasets to extract insights using mathematical techniques like data smoothing, correlation, and polynomial fitting.
4. develop and troubleshoot automated, efficient MATLAB scripts utilizing loops, decision branching, and custom functions.

Module	Title and Contents (Any 15 of the following)	Hrs.
Section I	1. Introduction to MATLAB: A Beginner's Guide	**60**
	2. MATLAB Commands	
	3. Basic Operations and Functions	
	4. Vectors and Matrices	
	5. Creating and Manipulating Arrays	
	6. Accessing Data in Arrays	
	7. Mathematical operations with arrays	
	8. Conditional data selection	
	9. Tables of data	
	10. Organizing tabular data	
	11. Data types	
	12. Preprocessing data	
	13. Data analysis tasks in MATLAB: Smoothing data	
	14. Data analysis tasks in MATLAB: Linear Correlation	

	15. Data analysis tasks in MATLAB: Fitting polynomials	
	16. Decision Branching	
	17. For Loops	
	18. While Loops	
	19. Increasing Automation with Functions	
	20. Troubleshooting code	

Reference Books:

1. A. Gilat., MATLAB: An Introduction with Applications. Wiley Publication. Fourth Edition 2012.
2. Stormy, MATLAB: A Practical Introduction to Programming and Problem Solving. Burlington, MA: Butterworth-Heinemann 2017.
3. P. Patankar and S. Kulkarni, MATLAB and Simulink In-Depth: Model-based Design with Simulink and Stateflow, User Interface, Scripting, Simulation, Visualization and Debugging. BPB Publications 2022.
4. R. J. Bansal, A. K. Goel and M. K. Sharma, MATLAB and its applications in Engineering. Pearsons Publication 2016.

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: SEC Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMPSEC 1 Mathematical Computations using Advanced Excel

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. learn basic mathematical operations in Excel.
2. explore mathematical data using advanced Excel tools.
3. analyze complex mathematical models in Excel.
4. emphasize mathematical concepts and Excel functionality to solve practical problems.

Course Outcomes: By the end of the course, students will be able to……

1. demonstrate proficiency in performing mathematical operations in Excel.
2. utilize Excel functions like AVERAGE, MEDIAN, and MODE to analyze numerical data sets, interpret the results, and draw conclusions.
3. create and manipulate formulas and functions, including nested functions and array formulas, to develop sophisticated mathematical models for various real-world applications.
4. integrate their understanding of mathematical principles with advanced Excel features.

Module	Title and Contents (Any 15 of the following)	Hrs.
Section I	1. Perform mathematical analysis on dataset using excel functions. (SUM, AVERAGE, MEDIAN, MODE etc.)	60
	2. Crete pivot tables to summarize and analyze data effectively	
	3. Absolute and Relative referencing	
	4. Perform simple Mathematical functions	
	5. Perform logical operations	
	6. Decision making using IF, SUMIF, COUNTIF etc.	
	7. Text functions and LOOKUP functions	
	8. Crete various types of graphs including line graphs, bar charts, pie charts	
	9. Explore various chart formatting options such as data labels and titles, colors, fonts and styles	
	10. Use of trend lines to identify and visualize trends within data sets.	
	11. Use of chart templates and annotations to highlights specific data points.	
	12. Combine different chart types.	
	13. Learn to create dynamic charts using Excel named ranges, and data validation for interactive data visualization.	
	14. Explore the creation of Histogram to visualize relationship between two continuous variables.	

	15. Create heat maps to visualize the density or intensity of data across different categories or dimensions using color gradients.	
	16. Create data models and relationships between multiple data set.	
	17. Use power query to clean, transform and shape data before visualizations.	
	18. Explore power pivot to create advanced calculations and measure.	
	19. Explore 3D mapping capabilities in Excel for visualizing geographical data.	
	20. Perform matrix calculations such as addition, subtraction, multiplication and inversion using Excel's array function.	

Reference Books:

1. Alan Murray, Advanced Excel Success: A Practical Guide to Mastering Excel, 1st Edition, Apress, 2020.
2. Jordan Goldmeier, Advanced Excel Essentials, 1st Edition, Apress, 2014.
3. L. Winston Wayne, Microsoft Excel 2019: Data Analysis & Business Model, PHI Learning Pvt. Ltd., 2019.
4. Naveen Mishra, Excel with Microsoft Excel: Comprehensive & Easy Guide to Learn Advanced MS Excel, Penman Books, 2019.
5. Ritu Arora, Mastering Advanced Excel, BPB Publication, 2023

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: IKS	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMTIKS 2 Vedic Mathematics

Course Objectives: By the end of the course, students should be able to……

1. explore the cultural, historical, and linguistic foundations of ancient Indian mathematical thought, from the Indus Valley through the Vedic period.
2. introduce the geometric principles, theorems, and spatial transformations documented in ancient texts like the Sulbasutras.
3. examine the historical evolution and mathematical significance of the decimal number system and the place-value principle.
4. trace the origins and development of foundational mathematical concepts—such as zero, infinity, arithmetic, and combinatorics—within Vedic literature.

Course Outcomes: By the end of the course, students will be able to……

1. explain the historical, cultural, and linguistic contexts that shaped early mathematical traditions in ancient India.
2. apply ancient Vedic geometric techniques (such as the *Sulbasutra* theorems) to perform rectilinear transformations and spatial constructions.
3. analyze the conceptual development of the decimal numeral system, various number bases, and the global impact of the place-value principle.
4. evaluate early numerical concepts, interpreting how infinity, zero, and early combinatorics were conceptualized and utilized in Vedic literature.

Module	Title and Contents	Hrs
Module - 1:	**Background: Culture and Language** 1.1 The Indus valley civilisation 1.2 The Vedic Period 1.3 The Oral tradition 1.4 Grammar.	07
Module - 2:	**Vedic Geometry** 2.1 The Sulbasutra 2.2 The theorem of the diagonal 2.3 Rectilinear Figures and their transformations 2.4 Circle from square: the direct construction 2.5 The inverse formula: Square from circle, generalities 2.6 Measures and Numbers, Geometry, Influences	08
Module - 3:	**Decimal Numbers** 3.1 Background, Numbers and Based numbers 3.2 The Place Value Principal and its Realisations	08

	3.3 Other Realisations 3.4 The Choice of a Base	
Module - 4:	**Numbers in the Vedic Literature** 4.1 Origin, Number names in Rigveda 4.2 Infinity and zero 4.3 Early Arithmetic 4.4 Combinatorics	07

Reference Books:

1. Datta and A. N. Sing, History of Hindu Mathematics, Reprint, Bharatiya Kala Prakashan, Delhi, 2004.
2. N. Srinivasiengar, History of Indian Mathematics, The World Press, Calcutta, 1967.
3. S. Seshadri, Studies in History of Indian Mathematics, Hindustan Book Agency, Delhi, 2010.
4. P. P. Divakaran, The Mathematics of India: Concepts, Methods, Connections, Hindustan Book Agency, 2018.Rep Springer New York, 2018.
5. Shri. B. K. Tirthaji, V. S. Agarwal, Vedic Mathematics or Sixteen Simple Mathematical formulae from Vedas, Orient Book Distributors,1981

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: Major	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 241 Real Analysis

Course Objectives: By the end of the course, students should be able to……

1. introduce the fundamental properties of convergent sequences of real numbers and the effects of algebraic operations on their limits.
2. establish the criteria for sequence convergence using the concepts of monotonicity, boundedness, and Cauchy sequences.
3. explain the concepts of limit superior and limit inferior and their relationship to the overall convergence of bounded sequences.
4. provide a comprehensive understanding of infinite series and equip students with various mathematical tests to determine convergence or divergence.

Course Outcomes: By the end of the course, students will be able to……

1. apply algebraic operations and limit theorems to calculate and evaluate convergent sequences of real numbers.
2. analyze the behavior of real number sequences to prove convergence or divergence using monotonic properties and Cauchy's criteria.
3. evaluate bounded sequences by computing their limit superior and limit inferior to mathematically deduce their overall convergence.
4. determine the absolute and conditional convergence of infinite series by selecting and executing appropriate mathematical tests (e.g., Comparison, Root, and Leibnitz tests).

Module	Title and Contents	Hrs
Module - 1:	**Operations on Convergent sequences** 1.1 Operation on Convergent sequence: 1.1.1 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n + t_n) = L + M$. 1.1.2 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n - t_n) = L - M$. 1.1.3 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, f $c \in \mathbb{R}$ and if $\lim_{n\to\infty} s_n = L$ then $\lim_{n\to\infty} cs_n = cL$. 1.1.4 Theorem: If $0 < x < 1$ then the sequence $\{x^n\}$ converges to 0. 1.1.5 Lemma: If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then $\{{s_n}^2\}_{n=1}^{\infty}$ converges to L^2. 1.1.6 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n.t_n) = LM$. 1.1.7 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if	7

	$\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n/t_n) = L/M$.	
Module - 2:	**Monotone Sequences and Cauchy Sequences** 2.1 Monotone Sequence 2.1.1 Definition and Examples 2.1.2 Theorem: A non-decreasing sequence which is bounded above is convergent. 2.1.3 Theorem: A non-increasing sequence which is bounded below is convergent. 2.1.4 Corollary: The sequence $\{(1+\frac{1}{n})^n\}$ is convergent. 2.1.5 Theorem (Without Proof): A non-decreasing sequence which is not bounded above diverges to infinity. 2.1.6 Theorem (Without Proof): A non-increasing sequence which is not bounded below diverges to infinity. 2.1.7 Theorem: A bounded sequence of real numbers has convergent subsequence. 2.2 Cauchy Sequence 2.2.1 Definition and Examples 2.2.2 Theorem: If sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence. 2.2.3 Theorem: If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is bounded. 2.2.4 Theorem: If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is convergent.	8
Module - 3:	**Limit Superior and Inferior of Sequences** 3.1 Definitions and Examples 3.1.1 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then $\lim_{n\to\infty} sup\, s_n = \lim_{n\to\infty} S_n$. 3.1.2 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then $\lim_{n\to\infty} inf\, S_n = \lim_{n\to\infty} S_n$. 3.1.3 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers and if $\lim_{n\to\infty} sup\, S_n = \lim_{n\to\infty} inf\, S_n = L$ where $L \in \mathbb{R}$ then $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n = L$. 3.1.4 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers and if $s_n \le t_n$ then $i)\ \lim_{n\to\infty} sup\, s_n \le \lim_{n\to\infty} sup\, t_n$ $ii)\ \lim_{n\to\infty} inf\, s_n \le \lim_{n\to\infty} inf\, t_n$. 3.1.5 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then $i)\ \lim_{n\to\infty} sup\,(s_n + t_n) \le \lim_{n\to\infty} sup\, s_n + \lim_{n\to\infty} sup\, t_n$ $ii)\ \lim_{n\to\infty} inf\,(s_n + t_n) \ge \lim_{n\to\infty} inf\, s_n + \lim_{n\to\infty} inf\, t_n$.	7
Module - 4:	**Series of Real Numbers** 4.1 Convergent and Divergent Series: 4.1.1 Definitions: Convergent Series, Divergent Series and Examples 4.1.2 If $\sum_{n=1}^{\infty} a_n$ is convergent series then $\lim_{n\to\infty} a_n = 0$. 4.2 Cauchy's General Principal for convergence (Statement only):	8

A necessary and sufficient condition for the convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that the sequence of its partial sum $\{s_n\}$ is convergent.

4.3 Series of Nonnegative real numbers:

4.3.1 Definition and Examples

4.3.2 Theorem: A positive term series converges if and only if its sequence of partial sum is bounded above.

4.3.3 Theorem: A positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $p > 1$.

4.4 Tests for convergence:

4.4.1 Comparison Test (First Type):

If $\sum u_n$ and $\sum v_n$ are two positive term series and $k \neq 0$, a fixed positive real number (independent of n) and there exists a positive integer m such that $u_n \leq k v_n$ for every $n \geq m$ then

$a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and

$b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent

4.4.2 Comparison Test (Second Type):

If $\sum u_n$ and $\sum v_n$ are two positive term series and there exist positive number m, such that $(u_n/u_{n+1}) \geq (v_n/v_{n+1})$ for every $n \geq m$ then

$a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and

$b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent

4.4.3 $p-$Series Test: The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

4.4.4 Root Test: Consider the series $\sum_{n=1}^{\infty} a_n$ .Then

$a)$ If $\lim sup|a_n|^{1/n} < 1$ then the series convergent absolutely

$b)$ If $\lim sup|a_n|^{1/n} > 1$ then the series diverges

$c)$ If $\lim sup|a_n|^{1/n} = 1$, this test gives no information

4.5 Alternating Series:

4.5.1 Leibnitz Test: If the alternating series

$u_1 - u_2 + u_3 - u_4 + \ldots$, ($u_n > 0$ for every n) is such that

$i)$ $u_{n+1} \leq u_n$, for every n and

$ii)$ $\lim u_n = 0$ then the series converges

4.6 Examples

4.7 Absolute and Conditional Convergence:

4.7.1 Definition and Examples

4.7.2 Theorem: Every Absolutely convergent series is convergent

4.8 Examples

Reference Books:

1. D. Somasundaram and B Choudhary, First Course in Mathematical Analysis, 8th Reprint, Narosa Publishing house New, Delhi, 2013
2. P. K. Jain and S. K. Kaushik, An Introduction to Real Analysis, 1st Edition, S. Chand & Company Ltd. New Delhi, 2000
3. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley India Pvt. Ltd, 2016

<table>
<tr><td colspan="2">4. Shanti Narayan and M. D. Raisinghania, Elements of Real Analysis, 15th Edition, S. Chand & Company Ltd. New Delhi, 2014</td></tr>
<tr><td colspan="2">Evaluation Pattern:</td></tr>
<tr><td colspan="2">Total Marks: 50</td></tr>
<tr><td>Internal Continuous Evaluation (20 Marks):
• CCE - I: 10 Marks: Assignment/Class Tests,
• CCE - II: 10 Marks: Case study/ Seminar/ Field Work
• CCE – III: 10 Marks: Multiple Choice questions
(The Best of two will be considered for the final mark entry)</td><td>End Semester Examination (30 Marks):
• Q.1) Solve the following questions. [5X2=10]
• Q.2) Attempt any two questions. [2X10=20]
• Q.3) Attempt any four questions. [4X5=20]
Note: Conversion of 50 marks of ESE evaluation to 30 marks.</td></tr>
</table>

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: Major	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 242 Abstract Algebra

Course Objectives: By the end of the course, students should be able to……

1. introduce the foundational concepts of abstract algebra, focusing on subgroups, cosets, and the application of Lagrange's theorem in finite groups.
2. explore the structural properties of cyclic groups, their generators, and their strong connections to number theory via Euler's and Fermat's theorems.
3. explain the significance of normal subgroups and how they are used to construct and understand quotient groups.
4. establish the principles of group mappings by teaching homomorphisms, fundamental isomorphism theorems, and permutation groups (Cayley's Theorem).

Course Outcomes: By the end of the course, students will be able to……

1. apply algebraic tests to identify subgroups and cosets, and utilize Lagrange's theorem to solve problems involving finite group structures.
2. analyze the properties of cyclic groups and compute their generators using the Euler phi function, Euler's theorem, and Fermat's theorem.
3. evaluate algebraic conditions to determine the normality of a subgroup and construct corresponding quotient groups.
4. prove structural relationships between different algebraic systems by applying homomorphisms, isomorphism theorems, and Cayley's theorem for permutation groups.

Module	Title and Contents	Hrs
Module - 1:	**Subgroups** 1.1 Subgroups 1.1.1 Definition: Subgroups with illustrations 1.2 Theorems on subgroup 1.2.1 Theorem: A non-empty subset H of a group G is a subgroup of G If and only if $a, b \in H \Rightarrow ab \in H$ and $a \in H \Rightarrow a^{-1} \in H$ 1.2.2 Theorem: A non-empty subset of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$ 1.2.3 Theorem: A non-empty finite subset H of a group G is a subgroup of G if and only if H is Closed under multiplication. 1.3 Centre of a Group 1.3.1 Definition: Centre of a Group, Normalizer of an element with illustration. 1.3.2 Theorem: Centre of group G is subgroup of group G. 1.3.3 Theorem: Normalizer of an element group G is subgroup of group G. 1.4 Cosets 1.4.1 Definition: Coset and examples	8

	1.4.2 Theorem: Let H be a subgroup of G then $i)\ Ha = H \Leftrightarrow a \in H\ and\ aH = H \Leftrightarrow a \in H$ $ii)\ Ha = Hb \Leftrightarrow ab^{-1} \in H$ and $aH = bH \Leftrightarrow a^{-1}b \in H$ $iii)\ Ha(aH)$ is a subgroup of G iff$a \in H$ 1.4.3 Theorem: $Ha = \{x \in G \mid x \equiv a\ mod\ H\} = cl(a)$for any a in G 1.4 Lagrange's Theorem 1.4.1 Theorem: If G is a finite group and H is a subgroup of G then $o(H)$ divides $o(G)$. 1.5 Index of a subgroup 1.5.1 Definition: Index of subgroup H in G with illustration 1.6 Theorem: For subgroups H and K of G, HK is a subgroup of G if and only if $HK = KH$	
Module - 2:	**Cyclic groups** 2.1 Cyclic groups 2.1.1 Definition: Cyclic group, generator of a cyclic group 2.1.2 Examples on 2.1.1 2.2 Theorems on Cyclic Groups 2.2.1 Theorem: Order of a cyclic group is equal to the order of its generator. 2.2.2 Theorem: A subgroup of cyclic group is cyclic. 2.2.3 Theorem: Every cyclic group is abelian. 2.2.4 Theorem: If G is finite group, then order of any element of G divides order of G. 2.2.5 Theorem: An infinite cyclic group has precisely two generators. 2.3 Euler ϕ function 2.3.1 Definition: Euler's $\emptyset$ function 2.3.2 Theorem: Number of generators of a finite cyclic group of order n is $\emptyset(n)$. 2.4 Euler and Fermat's Theorem 2.4.1 Euler's Theorem: Let $a, n(n \geq 1)$ be any integers such that $\gcd(a, n) = 1$ then $a^{\emptyset(n)} \equiv 1(\mathrm{mod} n)$ 2.4.2 Fermat's Theorem: For any integer a and prime p , $a^p \equiv a(\mathrm{mod} p)$ 2.4.3 Examples on 2.4.1 and 2.4.3	7
Module - 3:	**Normal groups** 3.1 Normal groups 3.1.1 Definitions: Normal subgroups, Simple group 3.1.2 Examples 3.2 Results on Normal Groups 3.2.1 Theorem: A subgroup H of group G is normal in G if and only if $g^{-1}Hg = H, g \in G$. 3.2.2 Theorem: A subgroup H of group G is normal in G if and only if $g^{-1}hg \in H$ for all $h \in H, g \in G$. 3.2.3 Theorem: A subgroup H of group G is normal in G if and only if the product of two right (left) cosets of H in G is again a right (left) coset of H in G.	7

	3.3 Quotient groups 3.3.1 Definition: Quotient groups with illustration. 3.3.2 Theorem: If G is finite group and N is normal subgroup of G then \ $o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$. 3.3.2 Theorem: Every quotient group of cyclic groups is cyclic.	
Module - 4:	**Homomorphism, Permutation Group** 4.1 Homomorphism 4.1.1 Definitions: Homomorphism, Epimorphism, Monomorphism, Endomorphism and Automorphism. 4.1.2 Examples on 4.4.1 4.1.3 Theorem: If f: $G \to G'$ is homomorphism then (i) $f(e) = e'$ (ii) $f(x^{-1}) = [f(x)]^{-1}$ (iii) $f(x^{n}) = [f(x)]^{n}$, n is an integer. 4.2 Kernel of Homomorphism 4.2.1 Definition: Kernel of Homomorphism with illustration 4.2.2 Theorem: If $f: G \to G'$ is homomorphism then kerf is a normal subgroup of G. 4.2.3 Theorem: A homomorphism $f: G \to G'$ is one-one if and only if $kerf = \{e\}$. 4.3 Isomorphism Theorems 4.3.1 Fundamental Theorem of group Homomorphism: If $f: G \to G'$ is an onto homomorphism with $K = kerf$ then $\frac{G}{K} \cong G'$. 4.3.2 Second theorem of Isomorphism :(Statement only) Let H and K be two subgroups of group G, where H is normal in G, then $\frac{HK}{H} \cong \frac{K}{H \cap K}$. 4.3.3 Third theorem of Isomorphism (Statement only): Let H and K be two normal subgroups of group G, such that $H \subseteq K$ then $\frac{G}{K} \cong \frac{G/H}{K/H}$. 4.4 Permutation Group 4.4.1 Cayley Theorem: Every group G is isomorphic to a permutation group. 4.4.2 Theorem: Set of even permutations is a normal subgroup of S_n Alternating group.	8

Reference Books:

1. A. R. Vasishtha, Modern Algebra, 2023rd Edition, Krishna Prakashan, Meerut, 2023.
2. I.N. Herstein, Topics in Algebra, 2nd Edition, Wiley India Pvt. Ltd, 2006.
3. J. B. Fraleigh, A first course in Abstract Algebra, 10th Reprint, Narosa Publishing House New Delhi, 2003.
4. V. K. Khanna and S. K. Bhambri, A course in abstract Algebra, 3rd Edition, Vikas Publishing house Private Limited, New Delhi, 2008

Evaluation Pattern:	
Total Marks: 50	
Internal Continuous Evaluation (20 Marks): • CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	**End Semester Examination (30 Marks):** • Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: Major Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMP 243 Practical Based on BMT 241 & BMT 242

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. introduce foundational concepts of abstract algebra, including subgroups, cyclic groups, and applications of Euler's and Fermat's theorems.
2. explore the structural properties of complex algebraic systems such as normal subgroups, quotient groups, permutations, and homomorphisms.
3. provide a rigorous understanding of the behavior of real number sequences, including convergence, monotonicity, limit superior, and limit inferior.
4. equip students with mathematical tests (Comparison, Root, Ratio) to evaluate the convergence or divergence of infinite series.

Course Outcomes: By the end of the course, students will be able to……

1. apply the properties of subgroups, cyclic groups, and number theory theorems (Euler/Fermat) to solve algebraic problems.
2. analyze complex algebraic relationships by examining normal subgroups, quotient groups, permutation groups, and homomorphisms.
3. evaluate the mathematical behavior of real number sequences by calculating limits, determining bounds, and proving convergence or monotonicity.
4. determine the absolute and conditional convergence of infinite series by selecting and applying the appropriate convergence tests.

Module	Title and Contents	Hrs
Section I	1. Subgroups	60
	2. Cyclic Group	
	3. Euler's Phi function and Fermat's theorem	
	4. Normal Subgroup	
	5. Quotient group	
	6. Permutation Group	
	7. Homomorphism of Groups	
	8. Operations on convergent sequences of real numbers	
	9. Monotone sequences	
	10. Limit superior of sequence of real numbers	
	11. Limit inferior of sequence of real numbers	
	12. Comparison test	
	13. Cauchy's n^{th} Root test	
	14. D'Alembert Ratio test	

	15. Absolutely and Conditionally Convergent series	

Reference Books:

1. A. R. Vasishtha, Modern Algebra, 2023rd Edition, Krishna Prakashan, Meerut, 2023.
2. D. Somasundaram and B Choudhary, First Course in Mathematical Analysis, 8th Reprint, Narosa Publishing house New, Delhi, 2013
3. I. N. Herstein, Topics in Algebra, 2nd Edition, Wiley India Pvt. Ltd, 2006.
4. J. B. Fraleigh, A first course in Abstract Algebra, 10th Reprint, Narosa Publishing House New Delhi, 2003.
5. P. K. Jain and S. K. Kaushik, An Introduction to Real Analysis, 1st Edition, S. Chand & Company Ltd. New Delhi, 2000.
6. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley India Pvt. Ltd, 2016.
7. Shanti Narayan and M. D. Raisinghania, Elements of Real Analysis, 15th Edition, S. Chand & Company Ltd. New Delhi, 2014.
8. V. K. Khanna and S. K. Bhambri, A course in abstract Algebra, 3rd Edition, Vikas Publishing house Private Limited, New Delhi, 2008 (Unit 1,2,3,4).

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: Minor	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 244 Numerical Analysis

Course Objectives: By the end of the course, students should be able to……

1. introduce direct and iterative numerical methods for solving non-linear algebraic equations and systems of linear equations.
2. explain computational techniques for determining eigenvalues and utilizing polynomial interpolation formulas for data approximation.
3. equip students with numerical integration techniques for evaluating definite integrals when analytical methods are complex or impossible.
4. demonstrate foundational single-step numerical methods for approximating solutions to ordinary differential equations (ODEs).

Course Outcomes: By the end of the course, students will be able to……

1. solve non-linear equations and systems of linear equations by implementing algorithms like Newton-Raphson and Gauss-Seidel.
2. compute dominant eigenvalues using the Power method and estimate unknown data points using appropriate interpolation techniques.
3. evaluate complex definite integrals computationally by applying the Trapezoidal and Simpson's rules.
4. apply numerical techniques, such as Euler's and Runge-Kutta methods, to find approximate solutions for ordinary differential equations.

Module	Title and Contents	Hrs
Module - 1:	**Root Finding Methods and System of Equations** 1.1 Bisection Method 1.2 Secant Method 1.3 Newton-Raphson Method 1.4 Gauss Elimination Method 1.5 Gauss Jordan Method 1.6 Gauss Seidel Method	8
Module - 2:	**Eigenvalues, Eigenvectors and Interpolation** 2.1 Eigenvalues 2.2 Eigenvectors 2.3 Power Method 2.4 Newton's Forward Interpolation 2.5 Newton's Backward Interpolation 2.6 Lagrange Interpolation 2.7 Divided Difference Interpolation	7

Module - 3:	**Integration** 3.1 Trapezoidal Rule and examples 3.2 Simpson's $\left(\frac{1}{3}\right)^{rd}$ Rule and examples 3.3 Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule and examples	7
Module - 4:	**Ordinary Differential Equations** 4.1 Euler Method 4.2 Modified Euler Method 4.3 Runge-Kutta of Second and Fourth Order Method, Examples	8

Reference Books:

1. Bhupendra Singh, Numerical Analysis, 6th Edition, Pragati Prakashan, 2017.
2. Devi Prasad, An Introduction to Numerical Analysis, 3rd Edition, Narosa Publishing House, 2008.
3. K. Sankara Rao, Numerical Methods for Scientists and Engineers, 4th Edition, Prentice Hall of India, 2018.
4. S. S. Sastry, Introduction Methods of Numerical Analysis, Prentice Hall of India, 2012.

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: Minor	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMT 245 Discrete Mathematics

Course Objectives: By the end of the course, students should be able to……

1. introduce mathematical logic and formal rules of inference (such as Modus Ponens and Modus Tollens) used to determine the validity of arguments.
2. teach the structural properties, arithmetic operations, and conversion techniques across diverse computational number systems.
3. establish a strong foundation in graph theory, covering graph properties, matrix representations, and concepts of connectivity (walks, paths, circuits).
4. explore hierarchical data structures through trees and demonstrate network optimization techniques like Kruskal's algorithm.

Course Outcomes: By the end of the course, students will be able to……

1. evaluate the logical validity of mathematical propositions and complex arguments using formal rules of inference and contradictions.
2. compute arithmetic operations (addition/subtraction) and perform base conversions across binary, decimal, quintal, octal, and hexadecimal systems.
3. analyze network structures by categorizing graphs, determining vertex degrees, identifying paths/circuits, and constructing matrix representations.
4. apply tree properties and Kruskal's algorithm to construct minimal spanning trees and solve network optimization problems.

Module	Title and Contents	Hrs
Module - 1:	**Valid and Invalid Arguments** 1.1 Revision 1.2 Mathematical logic 1.3 Modus Ponens and Modus Tollens 1.4 Additional valid argument forms 1.5 Rules of inferences, Contradictions, and valid arguments	6
Module - 2:	**Number Systems** 2.1 Number systems 2.2 Addition and subtraction of binary, decimal, quintal, octal, hexadecimal number systems and their conversions	7
Module - 3:	**Graphs** 3.1 Definition, Basic properties, Examples 3.2 Special Graphs, Directed and undirected graphs 3.3 Concept of degree, Matrix representation of graphs 3.4 Walk, Trail, Path and Circuits	9

Module - 4:	**Trees** 4.1 Definition and examples of trees 4.2 Rooted trees 4.3 Binary trees and their properties 4.4 Spanning trees, Minimal spanning trees, Kruskal's Algorithm	8

Reference Books:

1. C. L. Liu and D. P. Mohapatra, Elements of Discrete Mathematics, 4th edition, McGraw Hill Education, 2017.
2. K. H. Rosen, Discrete Mathematics and its Application, McGraw Hill, 2002.
3. N. L. Biggs, Discrete Mathematics, 2nd edition, Oxford University Press, 2013.
4. R. Johnsonbaugh, Discrete Mathematics, 8th edition, Pearson, 2013.
5. R. M. Somasundaram, Discrete Mathematical Structures, PHI Learning Pvt. Ltd., 2003.
6. S. Lipschutz and M. Lipson, Discrete Mathematics, Schaum's Outlines Series, Tata McGraw Hill, 2017.
7. Susanna S., Discrete Mathematics with Applications, PWS Publishing Company, 1995.

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: Minor Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMP 246 Practical Based on BMT 244 & BMT 245

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. introduce computational methods for root-finding, solving linear systems, finding eigenvalues, and performing polynomial interpolation.
2. provide practical experience in approximating definite integrals and solving ordinary differential equations numerically.
3. build a foundation in mathematical logic, including evaluating the validity of arguments, applying rules of inference, and converting number systems.
4. teach the fundamentals of graph theory, matrix representations of networks, and algorithms for finding minimal spanning trees.

Course Outcomes: By the end of the course, students will be able to……

1. compute numerical solutions for algebraic equations, eigenvalues, and data interpolation using algorithms like the Power method and Newton/Lagrange formulas.
2. apply numerical techniques (such as Simpson's rules and Runge-Kutta methods) to approximate complex integrals and solve differential equations.
3. analyze mathematical propositions to determine the validity of arguments using formal rules of inference.
4. solve network and path optimization problems by translating graphs into matrix representations and executing Kruskal's algorithm.

Module	Title and Contents	Hrs
Section I	1. Examples on root finding methods	60
	2. Examples on system of equations	
	3. Examples on eigen values, eigen vectors and Power method	
	4. Examples on Newton's forward and backward interpolation	
	5. Examples on Lagrange interpolation and Divided difference interpolation	
	6. Examples on Trapezoidal rule, Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule and Simpson's $\left(\frac{3}{8}\right)^{th}$ rule	
	7. Examples on Eulers method and modified Eulers method.	
	8. Examples on second and fourth order Runge-Kutta method	
	9. Valid and Invalid arguments.	
	10. Rules of inferences.	
	11. Number systems and their conversion.	
	12. Degree of vertices and graph.	
	13. Matrix representation of graph.	
	14. Minimal Spanning trees.	

	15. Kruskal's Algorithm.	

Reference Books:

1. Bhupendra Singh, Numerical Analysis, 6th Edition, Pragati Prakashan, 2017.
2. L. Liu and D. P. Mohapatra, Elements of Discrete Mathematics, 4th edition, McGraw Hill Education, 2017.
3. Devi Prasad, An Introduction to Numerical Analysis, 3rd Edition, Narosa Publishing House, 2008.
4. J. H. Mathews, Numerical Methods for Mathematics, Science and Engineering, 2nd Edition, Prentice Hall of International, 1992.
5. K. H. Rosen, Discrete Mathematics and its Application, McGraw Hill, 2002.
6. K. Sankara Rao, Numerical Methods for Scientists and Engineers, 4th Edition, Prentice Hall of International, 2018.
7. N. L. Biggs, Discrete Mathematics, 2nd edition, Oxford University Press, 2013.
8. R. Johnsonbaugh, Discrete Mathematics, 8th edition, Pearson, 2013.
9. R. M. Somasundaram, Discrete Mathematical Structures, PHI Learning Pvt. Ltd., 2003.
10. S. Lipschutz and M. Lipson, Discrete Mathematics, Schaum's Outlines Series, Tata McGraw Hill, 2017.

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: VSC	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMPVSC 2 Mathematical Computations by using MATLAB

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. introduce techniques for 2D and 3D data visualization and applying descriptive and inferential statistics to interpret datasets.
2. provide hands-on experience in formulating and solving constrained and unconstrained optimization problems, as well as root-finding for non-linear equations.
3. demonstrate the use of mathematical software (like MATLAB) for symbolic algebra, unit conversion, and calculus operations (derivatives, integrals, Taylor series).
4. train students in implementing numerical methods to solve single, higher-order, and systems of ordinary differential equations (ODEs).

Course Outcomes: By the end of the course, students will be able to……

1. analyze statistical properties of datasets and relationships between variables using 2D/3D visual charts and hypothesis testing.
2. solve unconstrained and constrained optimization problems and nonlinear equations using algorithms like the bisection method and built-in MATLAB functions.
3. compute algebraic manipulations, function approximations, derivatives, and integrals using symbolic variables in computational software.
4. evaluate complex mathematical models by applying numerical techniques to solve single and higher-order systems of ordinary differential equations.

Module	Title and Contents (Any 15 of the following)	Hrs
Section I	1. Visualizing data in 2D	**60**
	2. Visualizing data in 3D	
	3. Curve fitting	
	4. Visualizing data: Types of Visualizations, Scatter plot, Histogram, Scatter plot with Histogram, Box Plot	
	5. Descriptive Statistics: Descriptive Statistics, Measures of Center, Measures of Spread, Measures of Shape, Calculate Multiple Statistics for Grouped Variables	
	6. Normal Distribution, Hypothesis testing	
	7. Solving an unconstrained optimization problem in MATLAB	
	8. Solving a constrained optimization problem in MATLAB	
	9. Mathematical expression with symbolic variable	
	10. Algebraic Manipulation and Simplification	
	11. Working with Units of Measurement	
	12. Finding Derivatives and integrals	

	13. Approximating Functions using Taylor Polynomials	
	14. Root of a function and root finding problems	
	15. Bisection method	
	16. MATLAB function $fzero$ to find roots.	
	17. MATLAB function $fsolve$ to solve systems of nonlinear equations.	
	18. Solving ordinary differential equations numerically	
	19. System of ordinary differential equations	
	20. System of higher order ordinary differential equations	

Reference Books:

1. A. Gilat, MATLAB: An Introduction with Applications. Wiley Publication. Fourth Edition, 2012.
2. Stormy, MATLAB: A Practical Introduction to Programming and Problem Solving. Burlington, MA: Butterworth-Heinemann, 2017.
3. P. Patankar and S. Kulkarni, MATLAB and Simulink In-Depth: Model-based Design with Simulink and Stateflow, User Interface, Scripting, Simulation, Visualization and Debugging. BPB Publications, 2022.
4. R. J. Bansal, A. K. Goel and M. K. Sharma, MATLAB and its applications in Engineering. Pearsons Publication, 2016.

Evaluation Pattern: (60:40)

Total Marks: 50

End Semester Examination:

Internal Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	• Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - III
Type: SEC Practical	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMPSEC 2 Data Visualization using Python

Practical Batch: 20 Students / Batch

Course Objectives: By the end of the course, students should be able to……

1. introduce the Python data visualization ecosystem, including library installation (Matplotlib) and data preparation techniques.
2. demonstrate the construction and interpretation of standard statistical graphics for representing trends, distributions, and correlations.
3. explore advanced charting techniques for visualizing complex, hierarchical, and high-dimensional datasets.
4. provide hands-on experience with specialized visualization methods for text, networks, spatial (geospatial maps), and 3D data.

Course Outcomes: By the end of the course, students will be able to……

1. prepare and format raw datasets effectively for visual analysis using Python libraries.
2. construct foundational statistical charts (e.g., histograms, scatter plots, box plots) to accurately illustrate data distributions and relationships.
3. analyze complex, multi-variable data structures by generating advanced visuals like heatmaps, treemaps, and parallel coordinates.
4. design specialized visual representations—such as choropleth maps, word clouds, and 3D models—to communicate diverse data insights effectively.

Module	Title and Contents (Any 15 of the following)	Hrs.
Section I	1. Introduction to various Data Visualization tools in Python	60
	2. Basic Visualization in Python	
	3. Introduction to Matplotlib and Installation	
	4. Connecting to Data and preparing data for visualization in Matplotlib	
	5. Line Chart: To display trends over time	
	6. Bar Chart: To compare categories or groups	
	7. Histogram: Visualizing distribution of continuous data	
	8. Scatter Plot: Showing relationships between two variables	
	9. Pie Chart: Representing proportions of a whole	
	10. Box Plot: Displaying the distribution of data along a five-number summary	
	11. Heatmap: Visualizing data density or correlations	
	12. Violin Plot: Combining the features of a box plot and a kernel density plot	
	13. Bubble Chart: Displaying data points with varying sizes	
	14. Treemap: Representing hierarchical data using nested rectangles	

	15. Stacked Area Chart: Showing trends over time for multiple categories	
	16. Parallel Coordinates: Visualizing high-dimensional data	
	17. Network Graph: Displaying relationships between entities	
	18. Choropleth Map: Representing data values on a map	
	19. Word Cloud: Visualizing text data by displaying frequently occurring words	
	20. Explore 3D mapping in Python	

Reference Books:

1. Bharti Motwani, Data Analytics using Python, Wiley, 2020.
2. Dr. Abhinav, Data Visualization using Python Programming- - A Technical Guide for Beginners, Researchers and Data Analyst, Shashwat Publication, 2023.
3. Kalilur Rahman, Python Data Visualization Essentials Guide, BPB Online, 2021.
4. Seema Acharya, Reimagining Data Visualization Using Python, Wiley, 2022.
5. Swapnil Saurav, Learn and Practice Data Visualization using Python, 1st Edition, Eka Publishers, 2020.

Evaluation Pattern: (60:40)	
Total Marks: 50	
End Semester Examination:	
Internal Evaluation (20 Marks): • Journal: 10 Marks • Viva: 05 Marks • Student Performance: 05 Marks	**End Semester Examination (30 Marks):** • Q.1) Answer the following questions. (Any Two) [2 X 10 = 20] • Q.2) Answer the following questions. (Any Two) [2 X 05 = 10]

Karmaveer Bhaurao Patil University, Satara

(A State Public University Est. u/s 3(6) of MPUA 2016)

Faculty of Science and Technology

Yashavantrao Chavan Institute of Science, Satara

Board of Studies in Mathematics

Programme: B.Sc.	**Semester** - IV
Type: VEC	**Marks:** 50
Credits: 2	**From:** A. Y. 2025-26

Name of the Course: BMTVEC 2 Environmental Awareness with Mathematics

Course Objectives: By the end of the course, students should be able to……

1. explore the causes and consequences of global environmental crises while introducing mathematics as a problem-solving tool.
2. examine key environmental legislations, ethical frameworks, and the transition strategies necessary for sustainable development.
3. provide a comprehensive understanding of the Sustainable Development Goals (SDGs) and the frameworks used to track them globally and in India.
4. equip students with mathematical modeling and data analysis techniques for optimizing resources and mitigating environmental impact.

Course Outcomes: By the end of the course, students will be able to……

1. analyze the root causes and impacts of critical environmental issues, including climate change and resource depletion.
2. evaluate the effectiveness of environmental laws and ethical practices in guiding sustainable civic responsibilities.
3. examine the Sustainable Development Goals (SDGs) and utilize national and global indicator frameworks to track sustainability progress.
4. apply mathematical tools and ecological models to maximize resource efficiency, minimize waste, and calculate carbon footprints.

Module	Title and Contents	Hrs
Module - 1:	**Environmental Issues** 1.1 Pollution (Air, Water and Land) 1.2 Fresh-water overuse, Natural disasters 1.3 Fuel and Energy shortage due to overuse 1.4 Increase in wasteland, Biodiversity loss 1.5 Global warming and climate change 1.6 Role of mathematics in resolving environmental issues	08
Module - 2:	**Environmental Laws and Ethics** 2.1 Environmental Protection Act 2.2 Wildlife Protection Act 2.3 Forest Conservation Act 2.4 Prevention and Control of Pollution Act (Air, Water and Land) 2.5 From unsustainable to sustainable development 2.6 Responsibilities of an Environmentally aware citizen	08
Module - 3:	**Sustainable Development Goals** 3.1 Overview of the Sustainable Development Goals	07

	3.2 Sustainable Development Goals in India 3.3 Global Indicator Framework 3.4 National Indicator Framework and Guidelines	
Module - 4:	**Role of Mathematics in Meeting Sustainable Development Goals** 4.1 Data Analysis and Environmental Modeling 4.2 Application of mathematical tools and techniques to maximize resource efficiency 4.3 Minimize waste and improve distribution of resources 4.4 Calculation of Carbon Foot printing (CC) 4.5 Mathematical Ecology	07

Reference Books:

1. Danial D. Chiras, Environmental Science, Jones & Bartlett Learning, 2016.
2. P. D. Kaushik & Ajit Kumar, Understanding Sustainable Development Goals (SDGs): Concept, Implementation, Challenges and Opportunities, Springer, 2019.
3. P. Leela Krishnan, Environmental Law in India, LexisNexis, 2020.
4. R. Rajagopalan, Environmental Studies: From Crisis to Sustainable Development, Oxford University Press, 2018.
5. Tony Norton, Eleanor H. Norton & Nicholas A. Booker, Mathematics and the Sustainable Development Goals: Solving Complex Problems, Springer, 2020.

Evaluation Pattern:

Total Marks: 50

Internal Continuous Evaluation (20 Marks):	**End Semester Examination (30 Marks):**
• CCE - I: 10 Marks: Assignment/Class Tests, • CCE - II: 10 Marks: Case study/ Seminar/ Field Work • CCE – III: 10 Marks: Multiple Choice questions (The Best of two will be considered for the final mark entry)	• Q.1) Solve the following questions. [5X2=10] • Q.2) Attempt any two questions. [2X10=20] • Q.3) Attempt any four questions. [4X5=20] Note: Conversion of 50 marks of ESE evaluation to 30 marks.